２０２　その２

(1) $\sqrt{3}\sin n\theta+\cos n\theta=2\sin\left(n\theta+\dfrac{\pi}{6}\right)$ より

$n\theta+\dfrac{\pi}{6}>\dfrac{\pi}{6}$ より　$\sin\left(n\theta+\dfrac{\pi}{6}\right)=0$ を満たすθで小さいものから順に数えてk番目のθをθ_kとおくから

$$n\theta_k+\frac{\pi}{6}=k\pi$$

$$\theta_k=\frac{\pi}{n}\left(k-\frac{1}{6}\right)\quad (k=1,2,3,\cdots\cdots,n)$$

………（答）

(2) $$n\cos\frac{\theta_n}{2}=n\cos\left(\frac{\pi}{2}-\frac{\pi}{12n}\right)=n\sin\frac{\pi}{12n}$$

$\alpha=\dfrac{\pi}{12n}$ とおくと　$n=\dfrac{\pi}{12\alpha}$,　$n\to\infty$ のとき $\alpha\to+0$

$$\lim_{n\to\infty}n\cos\frac{\theta_n}{n}=\lim_{\alpha\to+0}\frac{\pi}{12\alpha}\sin\alpha$$

$$=\lim_{\alpha\to+0}\left(\frac{\pi}{12}\cdot\frac{\sin\alpha}{\alpha}\right)$$

$$=\frac{\pi}{12}$$

………（答）

(3) $$\frac{1}{n}\sum_{k=1}^{n}\cos\frac{\theta_k}{2}=\frac{1}{n}\sum_{k=1}^{n}\cos\left(\frac{k}{2n}\pi-\frac{\pi}{12n}\right)$$

$$=\frac{1}{n}\sum_{k=1}^{n}\left(\cos\frac{k}{2n}\pi\cos\frac{\pi}{12n}+\sin\frac{k}{2n}\pi\sin\frac{\pi}{12n}\right)$$

$$=\cos\frac{\pi}{12n}\cdot\frac{1}{n}\sum_{k=1}^{n}\cos\frac{k}{2n}\pi+\sin\frac{\pi}{12n}\cdot\frac{1}{n}\sum_{k=1}^{n}\sin\frac{k}{2n}\pi$$

したがって

$$\lim_{n\to\infty}\frac{1}{n}\left(\cos\frac{\theta_1}{2}+\cos\frac{\theta_2}{2}+\cdots\cdots+\cos\frac{\theta_n}{2}\right)$$

$$=\lim_{n\to\infty}\cos\frac{\pi}{12n}\left(\frac{1}{n}\sum_{k=1}^{n}\cos\frac{k}{2n}\pi\right)+\lim_{n\to\infty}\sin\frac{\pi}{12n}\left(\frac{1}{n}\sum_{k=1}^{n}\sin\frac{k}{2n}\pi\right)$$

$$=1\times\int_0^1\cos\frac{\pi}{2}x\,dx+0\times\int_0^1\sin\frac{\pi}{2}x\,dx$$

$$=\left[\frac{2}{\pi}\sin\frac{\pi}{2}x\right]_0^1=\frac{2}{\pi}$$

………（答）